

# P-V Criticality in Conformal Gravity holography in four Dimensions

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## Abstract

In this work we study the  $P - V$  criticality of conformal gravity(CG) holography in four dimensions by considering the cosmological constant as thermodynamic pressure. The main potential point of interest in CG is that there exists a *Rindler parameter*  $a$  in the metric. In the limit  $a = 0$ , one obtains the Schwarzschild-AdS black hole (BH). We investigate the thermodynamic behavior in the extended phase space and to examine what effects manifested in the *equation of state and critical constants* for this BH *due to the presence of Rindler term*  $a$ . We speculate that due to the presence of the said parameter there has been a deformation of the shape of the  $P - V$  diagram in comparison with RN-AdS BH and chargeless-AdS BH. Interestingly, we find the *critical ratio* <sup>1</sup> for this BH is  $\rho_c = \frac{P_c v_c}{T_c} = \frac{\sqrt{3}}{2} (3\sqrt{2} - 2\sqrt{3})$ , which is greater than the charged AdS BH and Schwarzschild-AdS BH that implies  $\rho_c^{CG} : \rho_c^{Sch-AdS} : \rho_c^{RN-AdS} = 0.67 : 0.50 : 0.37$ . The symbols are defined in the main work. Moreover, we observe that *critical constant is independent of the Rindler parameter*. Finally, we derive the *reduced equation of state* in terms of *reduced temperature, reduced volume and reduced pressure* respectively.

## 1 Introduction

The investigation of the thermodynamic properties of BHs in the AdS space has gained much more attention in recent years due to the famous work of Hawking and Page[1]. The AdS case is particularly interesting because of gauge-gravity duality via dual conformal field theory(CFT). The another striking feature of liquid-gas system was investigated by Chamblin et al. [3, 4, 5] for spherically symmetric charged AdS BH and they showed there should be exist the first order phase transition for Reissner-Nordström-AdS(RN-AdS) BH. The critical behavior of this BH has been studied there also and showed this behavior is analogous to the Van der Waals liquid-gas phase transition.

An active area of research in recent years by considering the cosmological constant as thermodynamic pressure [6, 7, 8, 9], ADM mass of the AdS BH as enthalpy of the thermodynamic system and the thermodynamically conjugate quantity is a volume then one should study the thermodynamic behavior in the extended phase space. Then one should investigate the critical properties which is analogous to Van der Waals liquid-gas system. This thermodynamic properties for RN-AdS BH has been studied in detail by Kubizňák-Mann[11] by considering the extended phase-space analysis. They beautifully described the analogy between Van der Waals fluid-gas system and charged AdS BH. They have written the equation of state and determined the critical constants in comparison with the liquid system. Critical constants and Critical exponents were calculated and shown to coincide with those of the liquid-gas systems. The critical behavior in the extended phase space has been studied elaborately for different BHs[12, 13].

In this work, we wish to examine the  $P - V$  criticality for CG holography [14] in four dimensions(4D) by considering the cosmological constant as thermodynamic variable and its conjugate quantity as thermodynamic volume. We find the equation of state in terms of BH temperature and

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<sup>1</sup>The ratio  $\mathcal{K}_c = \frac{T_c}{P_c v_c}$  is sometimes called Kamerlingh Onnes ratio [27]. This ratio for CG BH is  $\frac{2}{\sqrt{3}(3\sqrt{2}-2\sqrt{3})}$ , for RN-AdS BH is  $\frac{8}{3}$  and Schwarzschild-AdS BH is 2. Thus  $\mathcal{K}_c^{CG} : \mathcal{K}_c^{Sch-AdS} : \mathcal{K}_c^{RN-AdS} = 1.49 : 2 : 2.66$  that is  $\mathcal{K}_c^{CG} < \mathcal{K}_c^{Sch-AdS} < \mathcal{K}_c^{RN-AdS}$ .

specific thermodynamic volume. At the critical point, we calculate the critical constants. It is shown that the critical ratio is constant which is different from RN-AdS BH. We also derive the first law of thermodynamics and Gibbs free energy. Finally, we derive the reduced equation of state.

Furthermore, we examine what is the role of Rindler parameter in  $P - V$  criticality of CG holography. What is the effect of this term in BH thermodynamic equation of state and critical constants. We show that three critical constants: critical pressure, critical temperature and critical volume are *depends on the Rindler parameter*. The equation of state also depends on the said parameter. Interestingly critical ratio is *independent of Rindler term*. Due to this parameter the shape of  $P - V$  diagram in CG is completely different from Schwarzschild-AdS spacetime and RN-AdS space-time.

The fact that CG is a fascinating theory of gravity has a *non-trivial Rindler parameter*. It is a theory of gravity with large distance [16]. Like other higher curvature theories, it is also a renormalizable theory having ghost [17, 18]. On the other hand, the Einstein's general theory of gravity has no ghost i.e. ghost free gravity but two loop non-renormalizable [19]. To explain galactic rotating curves without dark matter, Mannheim was first studied phenomenologically this theory[20]. It has been emerges as a counter term in AdS/CFT correspondence [21, 22]. In the quantum gravity context, CG has been studied by 't Hooft [23]. Maldacena [24] has shown that by imposing appropriate boundary condition it is possible to eliminate the ghost term. The most important features of this theory is that it depends only on (Lorentz) angles but not on distances. It should be noted that CG is a higher-derivative theory but the entropy obeys the area law [14]. The another feature of CG is that the AdS boundary condition is weaker than Starobinsky boundary condition [15].

The organization of the paper is as follows. In Sec.(2), we have described the thermodynamic properties of CG holography in four dimensions. Finally, we give conclusion in Sec.(3).

## 2 Thermodynamics of CG Holography in Four Dimensions:

The CG theory is invariant under Weyl transformations  $g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu}$  where  $g_{\mu\nu}$  is the metric tensor and  $\Omega(x)$  is a function on space-time. Now the action for this theory is given by

$$\mathcal{I} = \int \sqrt{-g} d^4x C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta}. \quad (1)$$

where  $C_{\alpha\beta\gamma\delta}$  is the Weyl tensor. The equation of motion is derived by the Bach equation:

$$2\nabla_\alpha \nabla_\delta C^\alpha{}_{\beta\gamma}{}^\delta + C^\alpha{}_{\beta\gamma}{}^\delta R_{\alpha\delta} = 0. \quad (2)$$

where  $R_{\alpha\beta}$  is the Ricci tensor. The general spherically symmetric solution of the above action is described by the metric[14]

$$ds^2 = -\mathcal{F}(r)dt^2 + \frac{dr^2}{\mathcal{F}(r)} + r^2 d\Omega_2^2. \quad (3)$$

where,

$$\mathcal{F}(r) = \sqrt{1 - 12aM} - \frac{2M}{r} + 2ar - \frac{\Lambda}{3}r^2. \quad (4)$$

and  $d\Omega_2^2$  is the metric on the unit sphere in two dimension. Where  $a$  is the Rindler parameter.

In the limit  $a = 0$ , one finds the Schwarzschild-AdS space-time. In the limit  $aM \ll 1$ , one obtains Grumiller space-time[16]. Let us now put  $-\frac{\Lambda}{3} = \frac{1}{\ell^2}$  for AdS case.

The BH event horizon  $r_+$  could be obtain by solving  $\mathcal{F}(r_+) = 0$  i.e.

$$r_+^3 + 2a\ell^2 r_+^2 + \sqrt{1 - 12aM}\ell^2 r_+ - 2M\ell^2 = 0. \quad (5)$$

By solving the above equation one can obtain the mass parameter in terms of the event horizon radius as

$$M = \frac{r_+}{2} \left[ \sqrt{1 - 3a^2 r_+^2 - \frac{6ar_+^3}{\ell^2}} - ar_+ + \frac{r_+^2}{\ell^2} \right]. \quad (6)$$

In the limit  $a = 0$ , we get the ADM mass for Schwarzschild-AdS BH. It is given by

$$M = \frac{r_+}{2} \left[ 1 + \frac{r_+^2}{\ell^2} \right]. \quad (7)$$

It indicates that the mass parameter is a function of event horizon radius and strictly increasing function. But for CG BH the mass parameter is a function of both event horizon radius and Rindler parameter. It seems that due to Rindler acceleration the mass function is strictly decreasing and it could be observed from the Fig. 1. In the presence of charge the mass function becomes

$$M = \frac{r_+}{2} \left[ 1 + \frac{Q^2}{r_+^2} + \frac{r_+^2}{\ell^2} \right]. \quad (8)$$

The BH temperature is given by

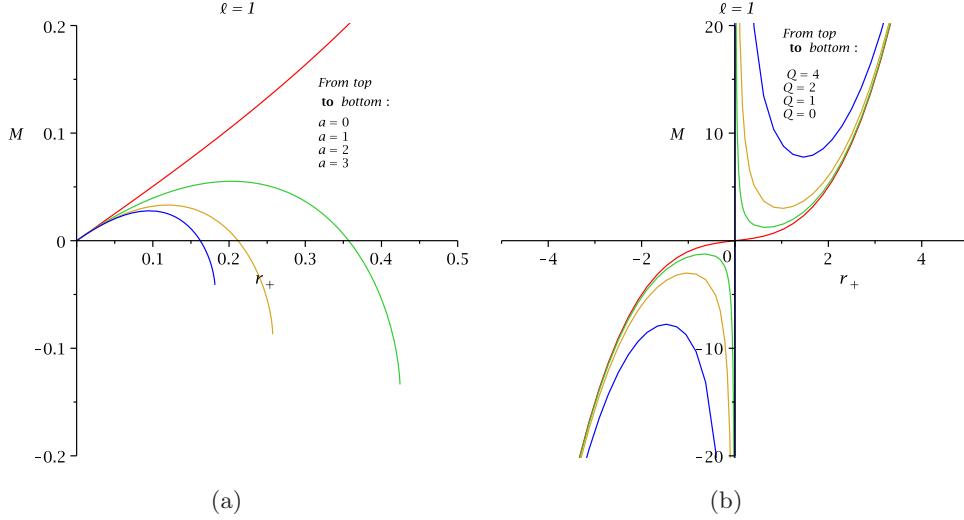


Figure 1: Variation of  $M$  with  $r_+$  for Schwarzschild-AdS BH, CG BH and RN-AdS space-time.

$$T = \frac{1}{4\pi r_+} \left( \sqrt{1 - 3a^2 r_+^2 - \frac{6a}{\ell^2} r_+^3 + a r_+ + 3 \frac{r_+^2}{\ell^2}} \right). \quad (9)$$

When  $a = 0$ , we find the temperature of famous Schwarzschild-AdS BH. The maximum and minimum value of the above temperature could be find from the following condition:

$$\left( \frac{\partial T}{\partial r_+} \right) |_{a,\ell} = 0. \quad (10)$$

which gives

$$54ar_+^7 + 36a^2\ell^2 r_+^6 - 9\ell^2 r_+^4 + 6a\ell^4 r_+^3 + \ell^6 = 0. \quad (11)$$

To find the exact root of this equation analytically it is not so trivial task rather we see what happens in the limit  $a = 0$ , we find

$$r_+ = \frac{\ell}{\sqrt{3}} = \frac{1}{\sqrt{8\pi P}}. \quad (12)$$

where the temperature is  $T = T_{min} = \frac{\sqrt{3}}{2\pi\ell} = \frac{0.275}{\ell}$  and a single BH is formed with horizon radius  $r_+$ . When  $T < T_{min}$ , there are no BHs but for  $T > T_{min}$ , there exists small and large BH their radii can

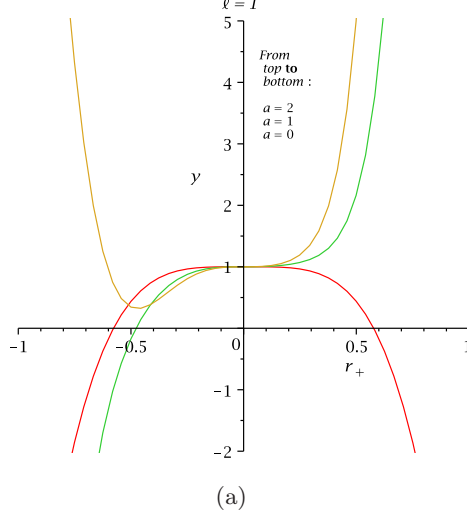


Figure 2: Variation of  $y = 54ar_+^7 + 36a^2\ell^2r_+^6 - 9\ell^2r_+^4 + 6a\ell^4r_+^3 + \ell^6$  with  $r_+$  for CG BH. For  $a = 0$ , the red one curve implies instability, whereas for  $a = 1, 2$  the green one and yellow one signals a stability.

be calculated from the Eq. 9 in the limit  $a = 0$ . We have plotted the Eq. 11 graphically to show the stability and instability region in Fig. 2.

By introducing charge parameter we get the temperature of RN-AdS BH. We do not write the explicit expression but our aim is to show the variation of this temperature with event horizon radius and compared it with our model. It could be found from the Fig. 3. The BH entropy should read

$$\mathcal{S} = \frac{\mathcal{A}}{4}. \quad (13)$$

where the area of the BH is

$$\mathcal{A} = 4\pi r_+^2. \quad (14)$$

Since in this work we are studying the  $P - V$  criticality in the extended phase space therefore one can define the cosmological constant as thermodynamic pressure and automatically its conjugate variable as thermodynamic volume:

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi\ell^2}. \quad (15)$$

and

$$V = \left( \frac{\partial M}{\partial P} \right)_S. \quad (16)$$

Now in the extended phase space, the mass parameter becomes

$$M = \frac{r_+}{2} \left[ \sqrt{1 - 3a^2r_+^2 - 16\pi aPr_+^3} - ar_+ + \frac{8\pi P}{3}r_+^2 \right]. \quad (17)$$

For this BH it should be

$$V = \left( \frac{\partial M}{\partial P} \right)_S = \frac{4}{3}\pi r_+^3 \left[ 1 - \frac{3ar_+}{\sqrt{1 - 3a^2r_+^2 - 16\pi P ar_+^3}} \right]. \quad (18)$$

It is quite strange that due to the *Rindler acceleration* the thermodynamic volume is get modified

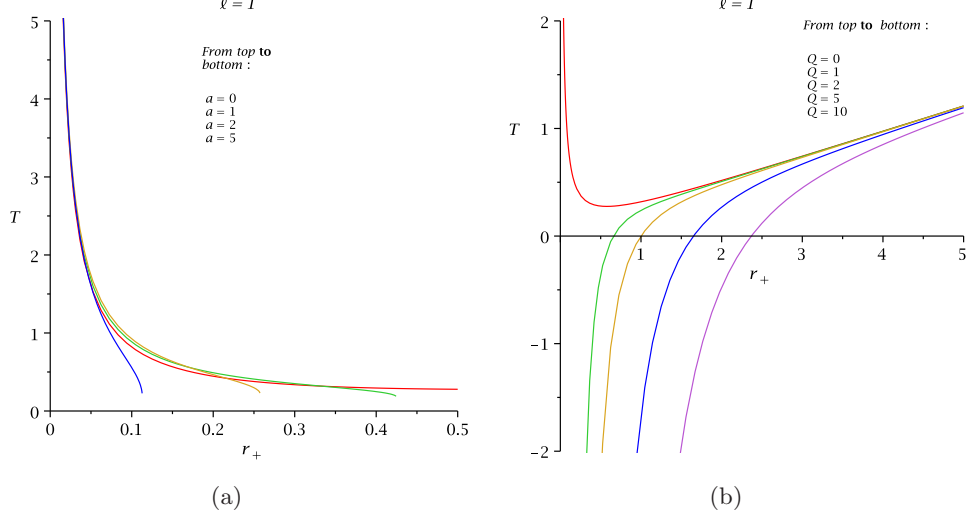


Figure 3: Variation of  $T$  with  $r_+$  for Schwarzschild-AdS BH, CG BH and RN-AdS BH. From the left figure it follows that the parameter  $a$  is modified the temperature variation in compared with Schwarzschild-AdS BH. In case of charged AdS BH it is completely different shape.

and it has been shown in Fig. 4. That means the thermodynamic volume depends upon the non-trivial Rindler parameter. This is the *first counter example* of any spherically symmetric BH that the thermodynamic volume is

$$V \neq \frac{4}{3}\pi r_+^3. \quad (19)$$

In general, we know that the thermodynamic volume for spherically symmetric BH ( For example for RN-AdS BH [11] ) is given by

$$V = \frac{4}{3}\pi r_+^3. \quad (20)$$

In Eq. 18, when Rindler parameter goes to zero value then we find the Eq. 20. The first law of thermodynamics should read

$$dM = TdS + VdP + \chi da. \quad (21)$$

where  $\chi$  is the physical quantity associated with the parameter  $a$ . It should be defined as

$$\chi = \left( \frac{\partial M}{\partial a} \right)_{S,P} = -\frac{r_+^2}{2} \left[ 1 + \frac{3r_+ \left( a + \frac{8\pi P}{3} r_+ \right)}{\sqrt{1 - 3a^2 r_+^2 - 16\pi a P r_+^3}} \right]. \quad (22)$$

Another novel feature of the thermodynamic volume is so called *Reverse Isoperimetric Inequality* [9] which is satisfied for all BHs except super-entropic BHs [10]. It has been conjecture that the thermodynamic volume  $V$  and the horizon area  $A$  is always satisfied the isoentropic ratio.

$$\mathcal{R} = \left( \frac{3V}{4\pi} \right)^{\frac{1}{3}} \left( \frac{4\pi}{\mathcal{A}} \right)^{\frac{1}{2}} \geq 1. \quad (23)$$

and for Schwarzschild-AdS BH, it is maximized. But interestingly, in our case this ratio is calculated to be

$$\mathcal{R} = \left[ 1 - \frac{3ar_+}{\sqrt{1 - 3a^2 r_+^2 - 16\pi P a r_+^3}} \right]^{\frac{1}{3}} \leq 1. \quad (24)$$

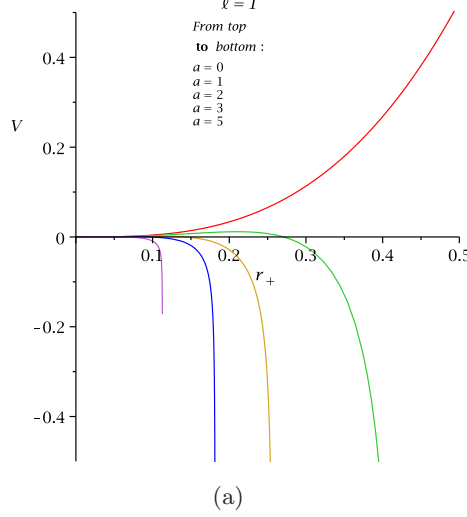


Figure 4: Variation of  $V$  with  $r_+$  for Schwarzschild-AdS BH and CG BH.

and found to be it is always *violate the Reverse Isoperimetric Inequality*. This is a second counter example (after [10]) of violation of the conjecture  $\mathcal{R} \geq 1$ . It could be seen from the Fig. 5. It should be noted that when  $a = 0$ , we obtain the ‘maximally entropic’ Schwarzschild-AdS BH [10].

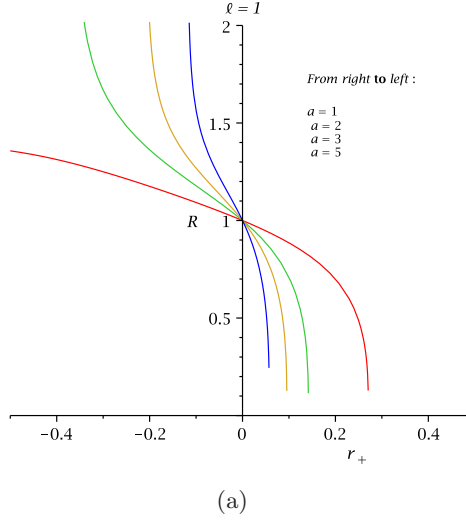


Figure 5: Variation of  $\mathcal{R}$  with  $r_+$  for CG BH for different values of Rindler parameter.

Finally the Gibbs free energy in the extended phase space should read

$$G = H - TS = \frac{r_+}{4} \left[ \sqrt{1 - 3a^2 r_+^2 - 16\pi P a r_+^3} - 3a r_+ - \frac{8\pi P}{3} r_+^2 \right]. \quad (25)$$

The Gibbs free energy and BH temperature both depends on the parameter  $a$ . The stability properties of small and large BHs could be determined by studying the features of  $G$ . From Eq. 25, it follows that when  $G = 0$ ,  $r_+ = 0$  that means the BH is in a pure radiation phase. The minimum value of the  $G$  is at the origin which suggests

$$r_+^4 - 4\pi T \ell^2 r_+^3 + (4\pi^2 T^2 + 12\pi a T + 12a^2) \ell^4 r_+^2 - \ell^4 = 0. \quad (26)$$

To find the exact numerical value of  $r_+$  from the above equation it is very difficult task. We can find the variation of this function with  $r_+$  for different values of  $a$  and  $T$  in graphically (See Fig 6).

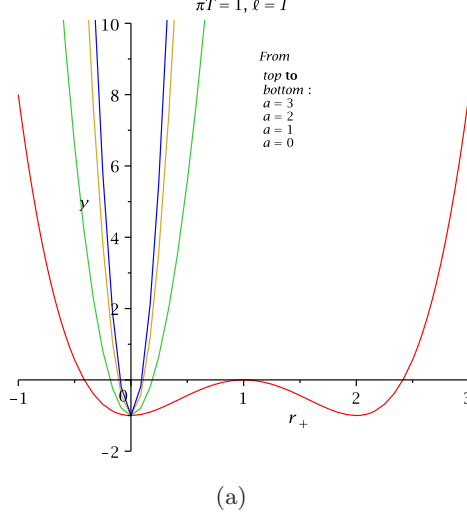


Figure 6: Variation of  $y = r_+^4 - 4\pi T \ell^2 r_+^3 + (4\pi^2 T^2 + 12\pi a T + 12a^2) \ell^4 r_+^2 - \ell^4$  with  $r_+$  for CG BH. It is clear from the figure due to the Rindler parameter HP transition for CG BH is modified.

But for  $a = 0$ , we can easily find the value of  $r_+$  and Hawking-Page phase transition temperature. From Eq. 26, we get

$$r_+^4 - 4\pi T \ell^2 r_+^3 + 4\pi^2 T^2 \ell^4 r_+^2 - \ell^4 = 0. \quad (27)$$

which is reduced to more simplify form :

$$(r_+^2 - 2\pi T \ell^2 r_+ + \ell^2) (r_+^2 - 2\pi T \ell^2 r_+ - \ell^2) = 0. \quad (28)$$

we discard the second one and from first one we find  $r_+ = \ell = \sqrt{\frac{3}{8\pi P}}$  when  $T = T_{HP} = \frac{1}{\pi \ell} = \sqrt{\frac{8P}{3\pi}}$ .

Where  $T_{HP}$  is called the famous Hawking-Page (HP) critical phase transition temperature [1]. For  $T > T_{HP}$ , the large BH is globally stable and for  $T < T_{HP}$ , the small BH is thermodynamically unstable, while the larger one is locally stable[25]. But in our case, due to the Rindler parameter we expect the  $T_{HP}$  is get modified by the factor  $a$ . We could not find the exact HP temperature due to the quartic nature of the equation, but we can say that it must be a function of Rindler parameter. In 7, we have drawn the Gibbs free energy for different values of temperature for CG BH. In 8, we have plotted the Gibbs free energy for different values of temperature for RN BH in comparison with CG BH.

Now we turn to the main work. Using Eqs. (9) and (15), the Hawking temperature could be rewritten as

$$T = \frac{1}{4\pi r_+} \left[ ar_+ + 8\pi P r_+^2 + \sqrt{1 - 3a^2 r_+^2 - 16\pi a P r_+^3} \right]. \quad (29)$$

Using this equation one can obtain the equation of state as

$$64\pi^2 r_+^4 P^2 + 32\pi r_+^3 (a - 2\pi T) P + (16\pi^2 r_+^2 T^2 - 8\pi a r_+^2 T + 4a^2 r_+^2 - 1) = 0. \quad (30)$$

This is a quadratic equation of  $P$ . Solving this equation one could find the *equation of state* for this AdS BH:

$$P = \frac{T}{2r_+} - \frac{a}{4\pi r_+} \pm \frac{\sqrt{1 - 8\pi a T r_+^2}}{8\pi r_+^2}. \quad (31)$$

It follows that from the above equation due to the presence of the Rindler term the *BH equation of state* is modified. First we consider the lower sign (the negative one) for  $P - V$  criticality. The  $P - V$

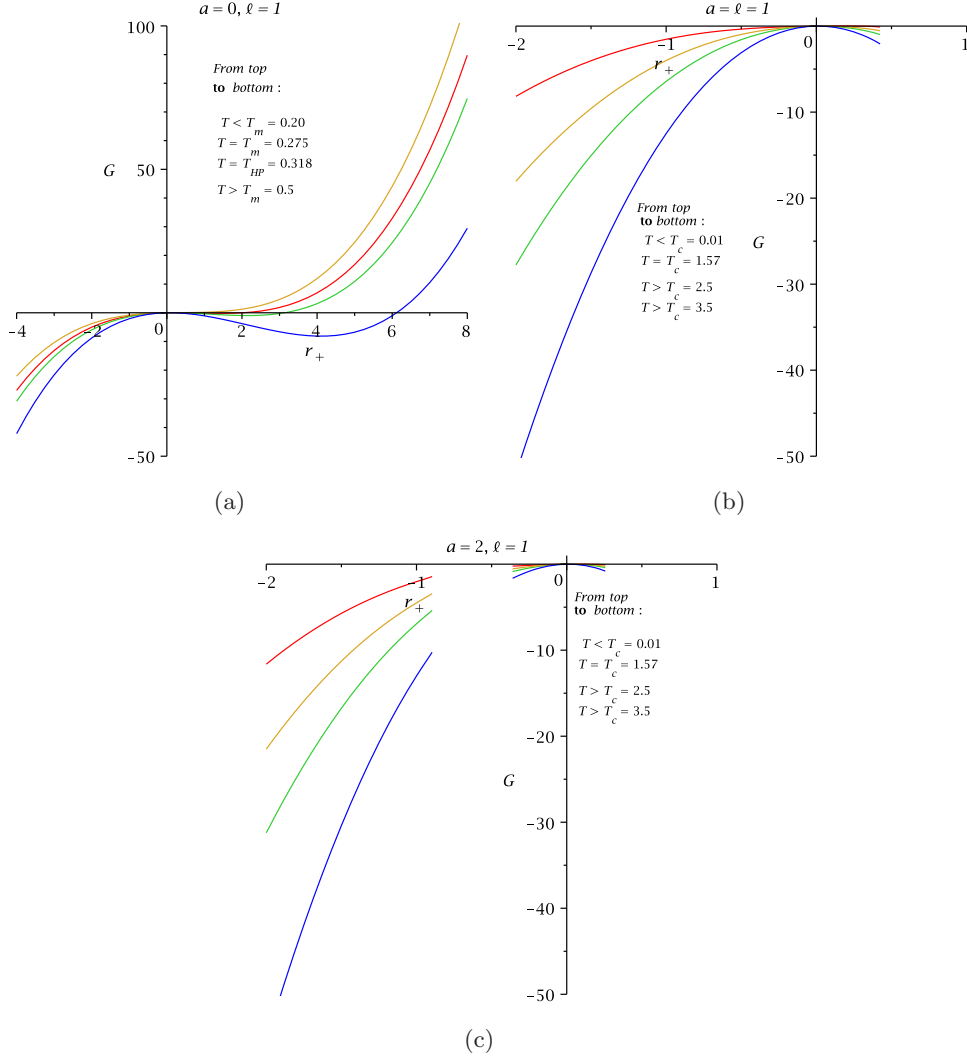


Figure 7: Variation of  $G$  with  $r_+$  for CG BH. It follows from the figure the variation of  $G$  with  $r_+$  without  $a$  and with  $a$  is qualitatively different.

criticality for upper sign should be considered in Appendix. In terms of specific volume  $v = 2r_+^2$ , one obtains the equation of state as

$$P = \frac{T}{v} - \frac{a}{2\pi v} - \frac{\sqrt{1 - 2\pi a v^2 T}}{2\pi v^2}. \quad (32)$$

In the limit  $a = 0$ , one has the equation of state for Schwarzschild-AdS BH [26]:

$$P = \frac{T}{v} - \frac{1}{2\pi v^2}. \quad (33)$$

where  $v = 2r_+ = 2\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$ .

The variation of  $P - r_+$  diagram can be seen from the Fig. 9-a. For each isotherm curve there corresponds to maximum value of  $P$  at  $r_+ = \frac{1}{2\pi T}$ . As we have mentioned earlier for a particular temperature  $T = T_{min}$ , the value of  $r_+ = r_{min} = \frac{1}{2\pi T_{min}} = \frac{\ell}{\sqrt{3}} = \frac{1}{\sqrt{8\pi P}}$ . It should be noted that  $G$  exhibits an inflection point at  $r_{min}$ . For greater value of this temperature we found the radii of large

<sup>2</sup>where  $r_+$  is the root of the equation  $16\pi a P r_+^9 + 12a^2 r_+^8 - (1 + 24a P V) r_+^6 - \left(\frac{9V}{2\pi}\right) a^2 r_+^5 + \left[\left(\frac{3V}{2\pi}\right) + \frac{9P V^2}{\pi}\right] r_+^3 + \left(\frac{27a^2 V^2}{16\pi^2}\right) r_+^2 - \left(\frac{3V}{4\pi}\right)^2 = 0$ , In the limit  $a = 0$ ,  $r_+ = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$ .



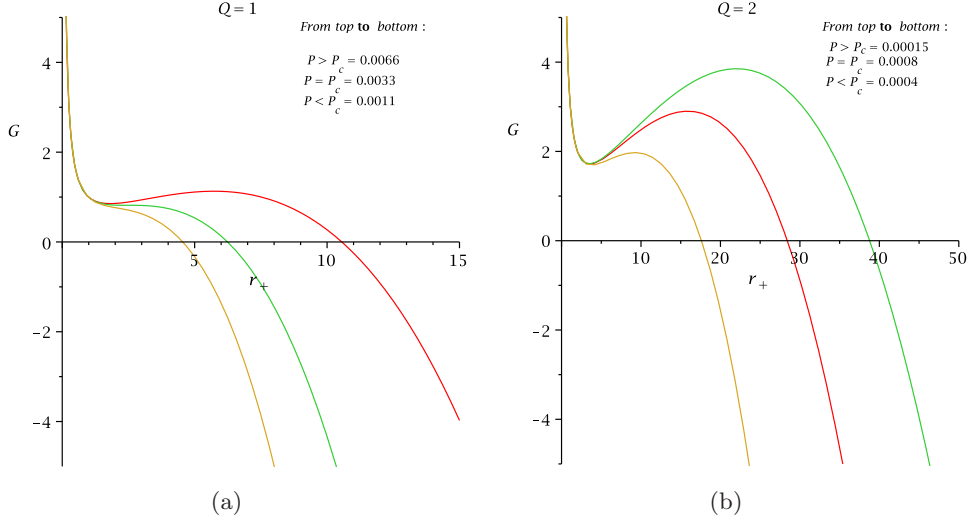


Figure 8: Variation of  $G$  with  $r_+$  for RN-AdS BH.

and small BH. From Eq. 30, we can obtain the radii of large and small BH for CG BH. Finding the exact root of the quartic equation numerically it is very difficult task, so we can plot this function graphically for various values of  $a$ ,  $P$  and  $T$  (See Fig. 10 ). But in the limit  $a = 0$ , the Eq. 30 reduces to the form:

$$64\pi^2 P^2 r_+^4 - 64\pi^2 T P r_+^3 + 16\pi^2 T^2 r_+^2 - 1 = 0. \quad (34)$$

which is more simplified form as:

$$(8\pi P r_+^2 - 4\pi T r_+ + 1) (8\pi P r_+^2 - 4\pi T r_+ - 1) = 0. \quad (35)$$

The first equation gives the radii of large and small Schwarzschild-AdS BH, is given by

$$r_{large} = \frac{T}{4P} \left[ 1 + \sqrt{1 - \frac{2P}{\pi T^2}} \right] \quad (36)$$

$$r_{small} = \frac{T}{4P} \left[ 1 - \sqrt{1 - \frac{2P}{\pi T^2}} \right] \quad (37)$$

where  $T_{min} = \sqrt{\frac{2P}{\pi}}$ . The second one gives another set of radii of large and small BH:

$$r_{large} = \frac{T}{4P} \left[ 1 + \sqrt{1 + \frac{2P}{\pi T^2}} \right] \quad (38)$$

$$r_{small} = \frac{T}{4P} \left[ 1 - \sqrt{1 + \frac{2P}{\pi T^2}} \right] \quad (39)$$

but which is unphysical because in the discriminant part the value of  $T$  gives imaginary value. Since we are not able to find the exact root of  $r_+$  due to quartic nature of Eq. 30, if we have set  $T = \frac{a}{2\pi}$  and assuming if it is the HP phase transition temperature for CG BH then the Eq. 30 reduces to the form:

$$64\pi^2 P^2 r_+^4 + 4a^2 r_+^2 - 1 = 0 \quad (40)$$

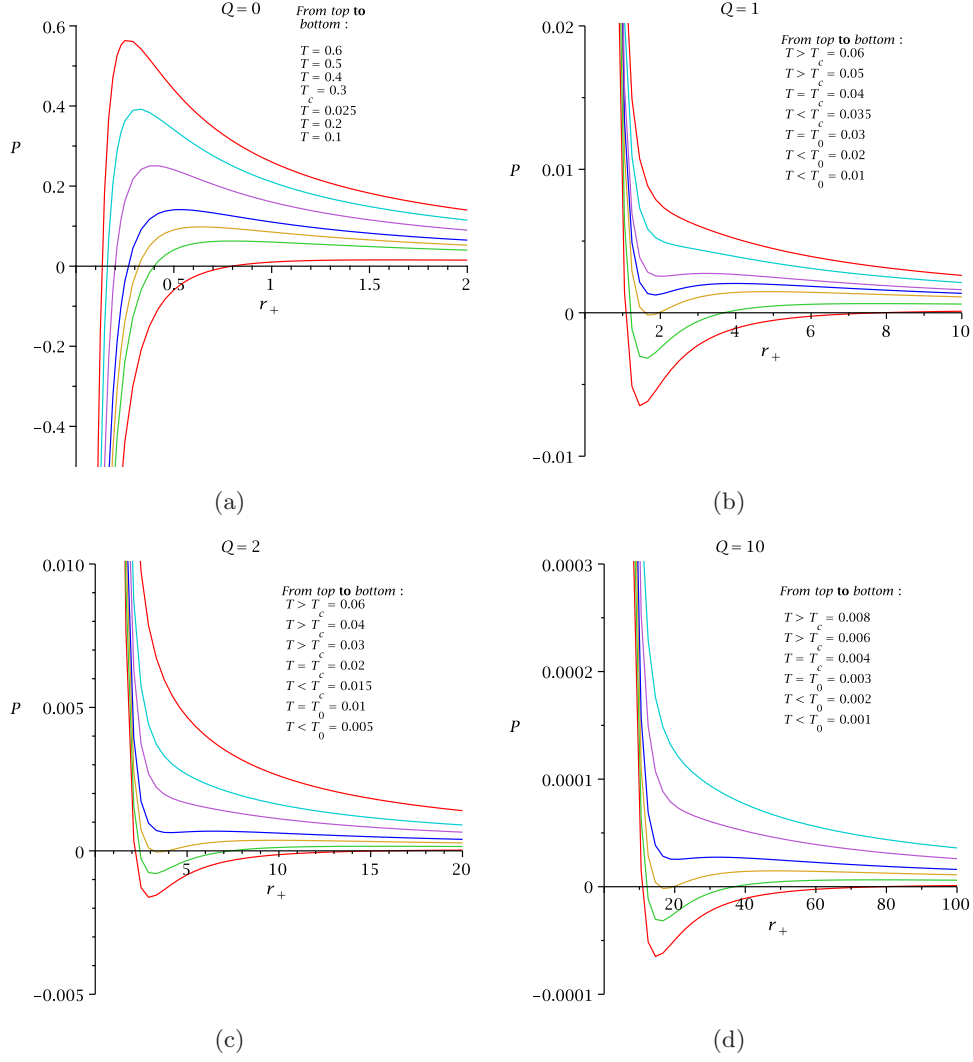


Figure 9: Variation of  $P$  with  $r_+$  for Schwarzschild-AdS BH and RN-AdS space-time.

then we have found the radii of large and small BH for CG gravity, given by

$$r_{large} = \sqrt{\frac{\sqrt{a^4 + 16\pi^2 P^2} - a^2}{32\pi^2 P^2}} \quad (41)$$

$$r_{small} = -\sqrt{\frac{\sqrt{a^4 + 16\pi^2 P^2} - a^2}{32\pi^2 P^2}} \quad (42)$$

so we can conclude that the HP temperature for CG BH must be of the type  $T_{HP} = f(a)$  that implies the HP temperature depends on Rindler parameter. The discussion of fluid analogue of Schwarzschild-AdS BH could be found in [26]. The critical constants could be determined by applying the following conditions at the inflection point:

$$\frac{\partial P}{\partial v}|_{T=T_c} = 0. \quad (43)$$

$$\frac{\partial^2 P}{\partial v^2}|_{T=T_c} = 0. \quad (44)$$

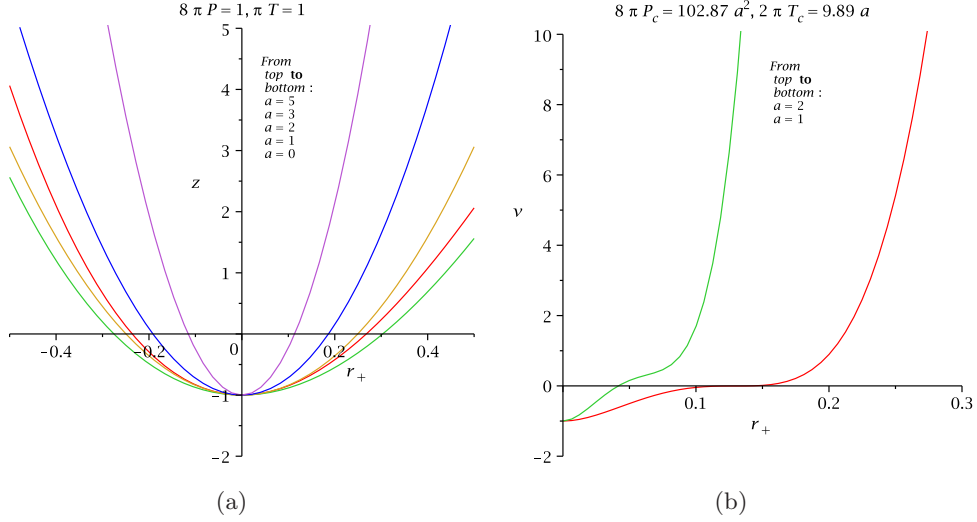


Figure 10: Variation of  $z = (8\pi P)^2 r_+^4 + 32\pi P (a - 2\pi T) r_+^3 + (16\pi^2 T^2 - 8\pi a T + 4a^2) r_+^2 - 1$  and  $v = (8\pi P_c)^2 r_+^4 + 32\pi P_c (a - 2\pi T_c) r_+^3 + (16\pi^2 T_c^2 - 8\pi a T_c + 4a^2) r_+^2 - 1$  with  $r_+$  for CG BH.

Solving Eq. (43), one can obtain

$$T_c = \frac{a}{2\pi} + \frac{1 - \pi a v_c^2 T_c}{\pi v_c \sqrt{1 - 2\pi a v_c^2 T_c}}. \quad (45)$$

Solving Eq. (44), one can find

$$T_c = \frac{a}{2\pi} + \frac{3 - 9\pi a v_c^2 T_c + 4\pi^2 a^2 v_c^4 T_c^2}{2\pi v_c (1 - 2\pi a v_c^2 T_c)^{3/2}}. \quad (46)$$

In the limit  $a = 0$ , one could obtain the critical constants of Schwarzschild-AdS BH:

$$P_c = \frac{1}{2\pi v_c^2} \quad (47)$$

$$T_c = \frac{1}{\pi v_c}. \quad (48)$$

If we choose  $v_c = 1$ , we find the critical values are  $P_c = \frac{1}{2\pi}$  and  $T_c = \frac{1}{\pi}$ . The interesting case happens in this spacetime is only HP phase transition [1] between large and small BHs. Witten [2] explained the HP phase transition is dual to the QCD confinement/deconfinement phase transition.

Using Eqs. (45) and (46), one has the critical Hawking temperature:

$$T_c = \frac{1}{3\pi a v_c^2}. \quad (49)$$

Using Eqs. (45) and (49), one obtains the critical volume:

$$v_c = \frac{3\sqrt{2} - 2\sqrt{3}}{3a}. \quad (50)$$

Finally, using Eqs. (32) and (49), we find the critical pressure:

$$P_c = \frac{\sqrt{3}}{2\pi v_c^2}. \quad (51)$$

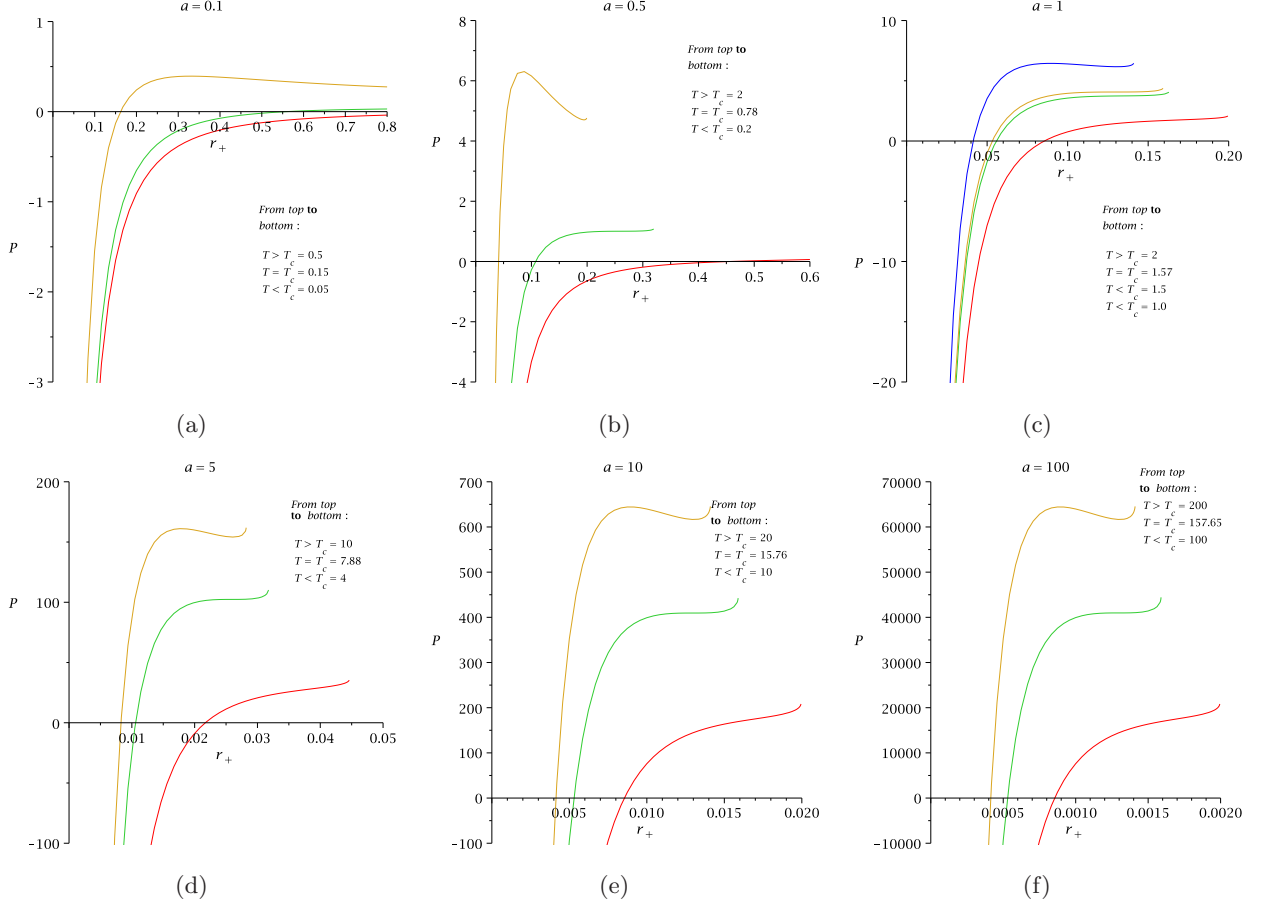


Figure 11: Variation of  $P$  with  $r_+$  for various values of  $a$ .

In terms of  $a$ , the critical values are

$$P_c = \frac{3\sqrt{3}}{4\pi(5-2\sqrt{6})}a^2 \quad (52)$$

$$v_c = \frac{3\sqrt{2}-2\sqrt{3}}{3a} \quad (53)$$

$$T_c = \frac{a}{2\pi(5-2\sqrt{6})}. \quad (54)$$

It may be noted that  $P_c$ ,  $v_c$  and  $T_c$  strictly depends upon the parameter  $a$ .

The critical ratio is given by

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{\sqrt{3}}{2} (3\sqrt{2} - 2\sqrt{3}). \quad (55)$$

interestingly it is independent of the Rindler parameter and which is a constant value as those find earlier for charged-AdS BH [11] given by

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}. \quad (56)$$

For Schwarzschild-AdS BH, the  $\rho_c$  is calculated to be

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{1}{2}. \quad (57)$$

Therefore the ratio of  $\rho_c$  for these BH should read

$$\rho_c^{CG} : \rho_c^{Sch-AdS} : \rho_c^{RN-AdS} = 0.67 : 0.50 : 0.37 . \quad (58)$$

It immediately follows that  $\rho_c^{CG} > \rho_c^{Sch-AdS} > \rho_c^{RN-AdS}$ . The “law of corresponding states” becomes

$$2\Theta = \left(3\sqrt{2} - 2\sqrt{3}\right) \Phi \left[ \sqrt{3}\Xi + \frac{\sqrt{1 - \frac{2}{3}\Theta\Phi^2}}{\Phi^2} \right] . \quad (59)$$

where  $\Theta$ ,  $\Phi$  and  $\Xi$  can be defined as

$$\Theta = \frac{T}{T_c} \quad (60)$$

$$\Phi = \frac{v}{v_c} \quad (61)$$

$$\Xi = \frac{P}{P_c} . \quad (62)$$

and these quantities like  $\Theta$ ,  $\Phi$  and  $\Xi$  are called *reduced temperature*, *reduced volume* and *reduced pressure* respectively. Thus the Eq. (59) is called the *reduced equation of state*.

### 3 Conclusion:

This work deals with the  $P - V$  criticality of CG holography in 4D by treating the cosmological constant as thermodynamic pressure. We have studied the thermodynamic properties in the extended phase space. The main potential point of interest in this work we have studied what is the key role of the *Rindler parameter*  $a$  in the extended phase space thermodynamics? We observed due to the said parameter there has been some effects manifested in the *BH thermodynamic equation of state and critical constants*. We also speculated that in the  $P - V$  diagram there has been a completely different shape in comparison with RN-AdS BH and Schwarzschild-AdS BH. Moreover, we have found that the critical ratio is independent of the Rindler parameter and it is satisfied the inequality  $\rho_c^{CG} > \rho_c^{Sch-AdS} > \rho_c^{RN-AdS}$ . Furthermore, we derived the reduced equation of state in terms of reduced temperature, reduced volume and reduced pressure respectively.

AppendixA:

In this appendix section we have examined the  $P - V$  criticality for the BH equation state of CG BH corresponds to

$$P = \frac{T}{2r_+} - \frac{a}{4\pi r_+} + \frac{\sqrt{1 - 8\pi a T r_+^2}}{8\pi r_+^2} . \quad (63)$$

As is in terms of specific volume  $v = 2r_+$ , the equation of state can be written as

$$P = \frac{T}{v} - \frac{a}{2\pi v} + \frac{\sqrt{1 - 2\pi a v^2 T}}{2\pi v^2} . \quad (64)$$

It should be noted that when  $a = 0$ , we do not find the BH equation of state for Schwarzschild-AdS BH. Yet, we should see what happens the change in the critical constants if we using the ‘+’ sign instead of ‘-’ sign. Doing all calculations as we have done previously, we find the critical constants are

$$P_c = -\frac{3\sqrt{3}}{4\pi(5 + 2\sqrt{6})}a^2 \quad (65)$$

$$v_c = \frac{3\sqrt{2} + 2\sqrt{3}}{3a} \quad (66)$$

$$T_c = \frac{a}{2\pi(5 + 2\sqrt{6})} . \quad (67)$$

It is strange that the critical pressure is negative. As usual the critical constants are depend on Rindler parameter. The critical ratio is given by

$$\rho_c = -\frac{\sqrt{3}}{2} \left( 3\sqrt{2} + 2\sqrt{3} \right) . \quad (68)$$

It is also curious result that the critical constant is negative.

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